Analytics Projects, Part 2
Data-Driven Prescriptive Analytics Projects

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Outline

Last time...

- Agenda:
  - Announcements:
    - Fill out the (new) **group assignment sheet** (link in the email)
    - Assignment due: **Reflection 1 (Nov 1, 23:59 AoE)**
    - Next assignment: **Midterm Report (due Nov 15)**, check rubrics at the course website
    - Office hour **tomorrow 8-9AM WIB** (Zoom link in MyITS).
  - Questions since our last class?
  - Discuss the group acvitity (Exercise 1.2)
  - Optimization Modeling Basics (Ch. 2)
  - Group activity

[Exercise 1.2] Discuss with your group

You are a sales manager tasked to increase sales for the upcoming quarter. You want to optimize allocation of the marketing budget to achieve this goal. You work with the marketing team, the sales team, the inventory management team, and the data and IT for the project.

1. The marketing team has historical data on the marketing budget allocation for all products, but they only use customer engagement metrics.
2. The sales team has data on the sales volume for each product item but only record sales if the product is in stock.
3. The inventory team has data on the inventory levels for each product item and the order received from the sales team, but not on lost sales.
4. The data and IT team maintains the data infrastructure and systems for the company. They can provide historical data and predictive models for sales volume with 100% accuracy!

Identify a proper scope for the project! What risks and obstacles you might face?

Optimization Modeling

- Model components:
  - Objective function $f(x)$: function to minimize (or maximize)
  - Decision variable x: parameter to optimize
  - Constraints $g(x) \leq 0$ and $h(x) = 0$: conditions for x to satisfy
  - Feasible set $\mathcal{X}$: set of all possible values for x
  - Parameters: other (static) values that affect the objective function and constraints
- Mathematical model:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g(x) \leq 0, \\ & h(x) = 0, \\ & x \in \mathcal{X} \end{aligned} \tag{1}$$

Example [Exercise 2.1]

- PLX is an electricity provider that is looking to optimize its service levels to maximize its profits. The company formulates the profit $f(x)$ (in millions of Rupiah) as a function of the service level x as

$$f(x) = -\frac{(x-95)^2 \cdot (x+100)^2}{1000} + 10000. \tag{2}$$

- The service level x is a continuous value between 0 and 100 (for this function to be valid).
  1. What is the **objective function**?
  2. What is the **decision variable**?
  3. What is the **constraints**?
  4. What are the **parameters**?
  5. What is the **optimal solution**?
  6. What is the **value of the optimal solution**?

Example [Exercise 2.1]

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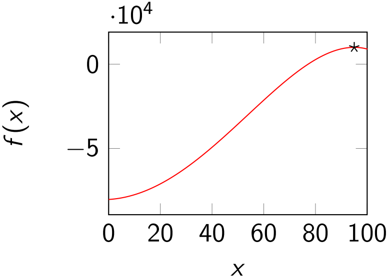
- The service level x is a continuous value between 0 and 100 (for this function to be valid).
  1. The **objective function**: $f(x) = -\frac{(x-95)^2 \cdot (x+100)^2}{1000} + 10000$
  2. The **decision variable**: x (service level)
  3. The **constraints**: $0 \leq x \leq 100$
  4. The **parameters**: $\{-95, +100, \frac{1}{1000}, +10000, \cdots\}$
  5. The **optimal solution**: $x^* = 95$ (why?)
  6. The **value of the optimal solution**: $f(x^*) = 10000$ (million Rupiah)

Identifying the optimal solution

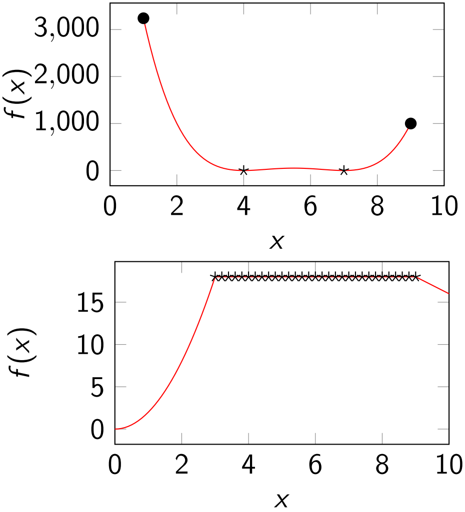
- The optimal solution x^* is the value of x that maximizes the objective function $f(x)$.
- The value of the optimal solution is $f(x^*)$.
- For the PLX example, we can find the optimal solution by:
  - Plotting the objective function $f(x)$
  - Finding the value of x that maximizes $f(x)$



  - The optimal solution is $x^* = 95$.
  - The value of the optimal solution is $f(x^*) = 10000$ (million Rupiah).

About the model and its optimal solution(s)

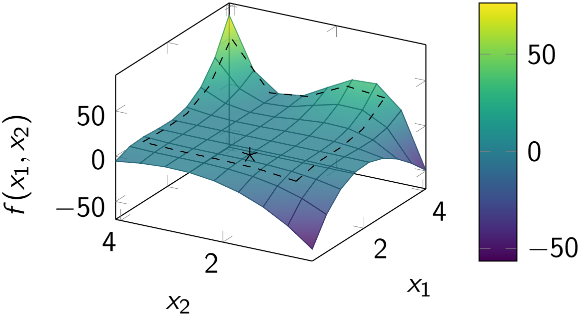
- We can have multiple optimal solutions.



About the model and its optimal solution(s)

- The decision variable can be multi-dimensional.

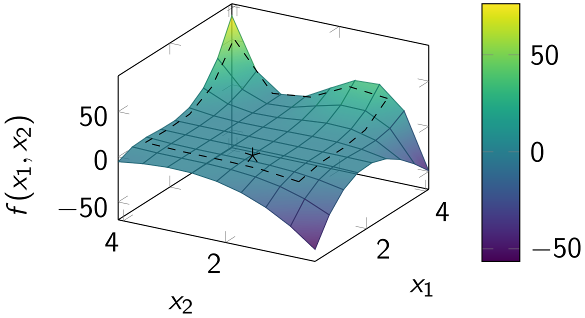
$$
\begin{aligned}
\underset{x_1, x_2}{\text{minimize}} \quad & (x_1 - 2)^2 \cdot (x_2 - 3)^2 \cdot \left(x_1 x_2 - \frac{5}{2} \right) \\
\text{subject to} \quad & 1 \le x_1 \le 4, \\
& 1 \le x_2 \le 4.
\end{aligned}
\tag{3}
$$



- $x_1^* = 1.438$, $x_2^* = 2.158$, $f(x_1^*, x_2^*) = 0.1350$.

About the model and its optimal solution(s)

- The decision variable can be multi-dimensional.



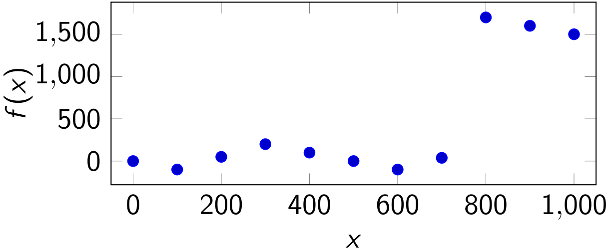
- There are values in the plot with lower objective values than the optimal solution. Why are they not optimal?

Sometimes the objective function itself is not fully known

- Example [Exercise 2.3]:

$$\begin{aligned} \underset{x}{\text{maximize}} \quad & f(x) = \text{CO}_2 \text{ capture economic value}(x) - x \\ \text{subject to} \quad & 0 \leq x \leq 1 \text{ billion Rupiah.} \end{aligned} \tag{4}$$

- $f(x)$ is only evaluated at $x \in \{0, 100, 200, \ldots, 1000\}$ million Rupiah.



- What is the optimal solution? If you are given more chances to evaluate $f(x)$ at some x's, which x values would you choose?

Summary

- When modeling an optimization problem, spend time to identify the objective function, decision variables, constraints, and parameters (and therefore the data you need).
- The optimal solution is the value of the decision variable that minimizes (or maximizes) the objective function.
- The optimal solution can be multi-dimensional, and there can be multiple optimal solutions.

Let's now take a look at some examples

- Example 1: PLX profit maximization
- Example 2: 2D optimization problem
- Example 3: Production planning

What's next?

- Next week:
  - Data Collection and Processing (Ch. 3)
  - Proposal Presentation (Group 1-2, random order)
- Next reading: Ch. 3 – Data Collection and Processing

Thank you!

- Questions?